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Ranking Triangular Intuitionistic Fuzzy Numbers: A Nagel Point Approach and Applications in Multi-Criteria Decision-Making

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Abstract

Ordinary fuzzy sets, first introduced by Zadeh in 1965 [1], have been expanded into several varieties, including type 2 fuzzy, intuitionistic fuzzy, hesitant fuzzy, and others, to assist us in modeling uncertainty. For every element in an Intuitionistic Fuzzy Set (IFS), there are membership and non-membership functions. Intuitionistic Fuzzy Numbers (IFNs) play a vital role in many applications. In this paper, a new ranking approach for IFNs was done based on Nagel points. The proposed new ranking was validated by certain results and numerical examples. In the last section, we put the suggested ranking principle into practice for the installation of an aeronautical research organization center through a case study.

Keywords: Intuitionistic fuzzy sets, Intuitionistic fuzzy number, Triangular intuitionistic fuzzy number, Nagel point, Multi-criteria decision-making problem.

1 | Introduction

Fuzzy numbers are real numbers with inaccurate precision that may deal with numerical imprecision. Generalizing the idea of fuzzy numbers, Intuitionistic Fuzzy Numbers (IFNs) are highly helpful in modeling situations with imprecise and insufficient data. The literature proposes a variety of IFNs for modeling real-world problems using different concepts. These include Real-Valued Intuitionistic Fuzzy Numbers (RVIFNs), Interval-Valued Intuitionistic Fuzzy Numbers (IVIFNs), Triangular Intuitionistic Fuzzy Numbers

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(TIFNs), and Conventional Trapezoidal Intuitionistic Fuzzy Numbers (CTrIFNs). Each variety of IFNs has a distinct benefit for modeling real-world scenarios, and they are all necessary. For applying suitable ranking procedures for real-time problems, it is necessary to order them.

A new parametric method for ranking IFNs was proposed, and its superiority over other methods was demonstrated using appropriate examples in [2]. Total ordering on the entire class of IFNs using upper lower dense sequence was proposed and compared with existing techniques using illustrative examples in [3]. A new ranking was proposed based on considering a fuzzy origin and measuring the distance of each IFN from the fuzzy origin in [4]. The method based on the incircle of the membership and non-membership functions of TIFNs uses lexicographical order to rank IFNs and was investigated by Atalik and Senturk in [5] in 2020. A new type-2 intuitionistic exponential triangular fuzzy number was introduced in [6], and some properties and theorems of this type of fuzzy number with graphical representations have been studied, and some examples are given to show the effectiveness of the proposed method. Also, the ranking function based on the centroid concept and Euclidean distance of the generalized exponential triangular IFNs is computed. The defuzzification measure based on the area of the region for intuitionistic triangular fuzzy numbers was introduced in 2019 [7]. Annie Christi and Kasthuri [8] applied a ranking technique called the accuracy function and Russell's Method to find the optimal solution of the transportation problem in 2015. Salahshour et al. [9] suggested a new defuzzification method for obtained TFNs using their values and ambiguities. The mean value of intuitionistic fuzzy variables is obtained using the credibility distribution. Then, a more comprehensive ranking method based on the mean value of intuitionistic fuzzy variables was developed in 2017 [10]. A new ranking method is developed on the basis of the concept of a ratio of the value index to the ambiguity index and applied to multiattribute decision-making problems in which the ratings of alternatives on attributes are expressed with TIFNs [11]. Ummusalma and Selvakumari [12] present a solution methodology for Multi-Criteria Decision-Making (MCDM) problems in an intuitionistic fuzzy environment in which the ratings of choice on attributes are represented by triangular IFNs. In 2014, Liang et al. [13] developed a method for ranking triangular intuitionistic fuzzy numbers based on score and accuracy functions. Later, it was applied to solve multi-criteria decision-making problems. In 2017, Stanujkic et al. [14] used Hamming distance to rank triangular IFNs, which was applied to a case study of hotels' website evaluation. An extended DEA model is proposed in a triangular intuitionistic fuzzy environment, where the inputs and outputs of DMUs are TIFNs [15].

A new ranking based on the preference degree of TFNs is defined by combining the probability and possibility of TFNs, which considers all the possible values in TFNs. Based on this ranking, a new method is proposed to solve the triangular fuzzy MAGDM problem. Wang [16] proposed a ranking algorithm for the TIF-MCDM issue for the accessible options by processing the various distances between the perfect option and all the accessible options [17]. An extension of the fuzzy Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) for MCDM problems in an intuitionistic fuzzy environment based on magnitude ranking was studied by Gautam et al. [18]. Selvaraj and Majumdar [19] proposed a non-membership score function which has been utilized in Interval-Valued Intuitionistic Fuzzy Technique for Order of Preference by Similarity to Ideal Solution (IVIF-TOPSIS) using numerical examples. For many decision problems with uncertainty, triangular IFNs are a useful tool in expressing ill-known quantities. Xu et al. [20] develop a novel decision method based on a zero-sum game for multiple-attribute decision-making problems where the attribute values take the form of TIFNs, and the attribute weights are unknown.

Novelty and motivation: The motivation behind research work in the ranking of IFNs lies in addressing the challenges posed by uncertainty and vagueness in decision-making processes. The ranking of IFNs becomes crucial in various applications, particularly in decision-making and optimization problems, where the decision-makers need to compare and prioritize alternatives under uncertainty. Here are some key motivations for research in this area:

I. Decision support systems: In real-world decision-making scenarios, decision-makers often deal with incomplete or imprecise information. The ranking of IFNs is essential for developing decision-support systems that can effectively handle uncertainty, thereby enabling better decision outcomes.

II. Optimization problems: Optimization problems under uncertainty require methodologies that can handle fuzzy or uncertain data. Ranking IFNs is a key step in developing optimization algorithms that can work with intuitionistic fuzzy information, leading to more robust and adaptable solutions.

III. Risk management: In various fields such as finance, engineering, and environmental science, decisions are often made in the presence of risk. IFNs can model uncertainty and risk more accurately than traditional methods. Research on ranking IFNs helps develop tools for risk assessment and management.

2 | Preliminaries

The definitions and ideas pertaining to Intuitionistic Fuzzy Sets (IFSs) and intuitionistic fuzzy numbers are briefly summarized in this section.

Definition 1 ([21]). An IFS A in X is given by $A = \{(x, \mu_A(x), \nu_A(x), x \in X)\}$, where the functions $\mu_A(x): X \to [0,1]$ and $\nu_A(x): X \to [0,1]$ define, respectively, the degree of membership and degree of non-membership of the element $x \in X$ to the set A, which is a subset of X, and for every $x \in X, 0 \leq \mu_A(x) + \nu_A(x) \leq 1$. Obviously, every fuzzy set has the form $\{(x, \mu_A(x), \mu_{A^c}(x)), x \in X\}$.

For each IFS A in X, we will call $\Pi_A(x) = 1 - \mu(x) - \nu(x)$ the intuitionistic fuzzy index of x in A. It is obvious that $0 \leq \Pi_A(x) \leq 1$, for all $x \in X$.

Definition 2 ([22]). An IFS $A = \{(x, \mu_A(x), \gamma_A(x) | x \in X)\}$ is called IF-normal if there exist at least two points $x_0, x_1 \in X$ such that $\mu_A(x_0) = 1, \gamma_A(x_1) = 1$. It is easily seen that a given IFS A is IF-normal if there is at least one point that surely belongs to A and at least one point that does not belong to A.

Definition 3 ([22]). An IFS $A = \{(x, \mu_A(x), \gamma_A(x) | x \in X)\}$ of the real line is called IF-convex if $\forall x_1, x_2 \in \mathbb{R}, \forall \lambda \in [0,1]$

$$\mu_A(\lambda x_1 + (1 - \lambda)x_2) \geq \mu_A(x_1) \wedge \mu_A(x_2),$$

$$\gamma_A(\lambda x_1 + (1 - \lambda)x_2) \geq \gamma_A(x_1) \wedge \gamma_A(x_2).$$

Thus, A is IF–convex if its membership function is fuzzy convex and its non-membership function is fuzzy concave.

Definition 4 ([22]). An IFS $A = \{(x, \mu_A(x), \gamma_A(x) | x \in X)\}$of the real line is called an IFN if

I. A is IF-normal,

II. A is IF-convex,

III. μ_A is upper semicontinuous and γ_A is lower semicontinuous,

IV. $A = \{(x \in X | \gamma_A(x) < 1\}$ is bounded.

Definition 5 ([23]). A TIFN is an IFS in R with the following membership function $\mu_A(x)$ and non-membership function $\vartheta_A(x)$

$$\mu_A(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, a_1 \leq x \leq a_2\,, \\ \dfrac{x - a_3}{a_2 - a_3}, a_2 \leq x \leq a_3, \\ 0, \qquad \text{otherwise}, \end{cases}$$

$$\vartheta_A(x) = \begin{cases} \dfrac{a_2 - x}{a_2 - a_1'}, a_1' \leq x \leq a_{2,} \\ \dfrac{x - a_2}{a_3' - a_2}, a_2 \leq x \leq a_{3,}' \\ 1, \qquad \text{otherwise}, \end{cases}$$

where $a_1' \leq a_1 \leq a_2 \leq a_3 \leq a_3'$ and $\mu_A(x) + \vartheta_A(x) \leq 1$ or $\mu_A(x) = \vartheta_A(x)$ for all $x \in R$. This TIFN is denoted by $A = (a_1, a_2, a_3; a_1', a_2, a_3')$.

Definition 6 ([23]). Let $A = (a_1, a_2, a_3; a_1', a_2, a_3')$ and $B = (b_1, b_2, b_3 b_1', b_2, b_3')$, then

I. $A \oplus B = \{(a_1 + b_1, a_2 + b_2, a_3 + b_3); (a_1' + b_1', a_2 + b_2, a_3' + b_3')\}$.

II. $A \otimes B = \begin{Bmatrix} (\min(a_1 b_1, a_1 b_3, a_3 b_1, a_3 b_3), a_2 b_2, \max(a_1 b_1, a_1 b_3, a_3 b_1, a_3 b)); \\ (\min(a_1' b_1', a' b_3', a_3' b_1', a_3' b_3'), a_2 b_2, \max(a_1' b_1', a' b_3', a_3' b_1', a_3' b_3')) \end{Bmatrix}$

III. $kA = (ka_1, ka_2, ka_3; ka_1', ka_2, ka_3')$.

3 | Ranking of Triangular Intuitionistic Fuzzy Numbers Using Nagel Point

This section describes the method for finding the coordinates of the Nagel point and the algorithm for ordering TIFNs using Nagel points.

Let $\tilde{A}_{TIFN} = (a_1, a_2, a_3; a_1', a_2, a_3')$ be a TIFN. The Cartesian coordinates that correspond to the vertices of two triangles ABC and DBE representing the membership function $\mu_{\tilde{A}}(x)$ and non-membership function $\nu_{\tilde{A}}(x)$ n are $A(a_1', 0), B(a_2, 0), C(a_3', 0), D(a_1, 0)$ and $E(a_3, 0)$.

Then

$$= |BC| = \sqrt{(a_2 - a_3')^2 + 1}, b = |CA| = a_3' - a_1', c = |BA| = \sqrt{(a_2 - a_1')^2 + 1}.$$

$$d = |BE| = \sqrt{(a_2 - a_3)^2 + 1}, e = |DE| = a_3 - a_1, f = |BD| = \sqrt{(a_2 - a_1)^2 + 1}.$$

The trilinear coordinates $\alpha:\beta:\gamma$ of the Nagel point $N_{1-\nu_{\tilde{A}}}$ of triangle ABC can be found as follows:

$$\alpha_{1-\nu_{\tilde{A}}} = \frac{b + c - a}{a} = \frac{a_3' - a_1' + \sqrt{(a_2 - a_1')^2 + 1} - \sqrt{(a_2 - a_3')^2 + 1}}{\sqrt{(a_2 - a_3')^2 + 1}}.$$

$$\beta_{1-\nu_{\tilde{A}}} = \frac{c + a - b}{b} = \frac{\sqrt{(a_2 - a_3')^2 + 1} + \sqrt{(a_2 - a_1')^2 + 1} - (a_3' - a_1')}{a_3' - a_1'}.$$

$$\gamma_{1-\nu_{\tilde{A}}} = \frac{a + b - c}{c} = \frac{\sqrt{(a_2 - a_3')^2 + 1} + a_3' - a_1' - \sqrt{(a_2 - a_1')^2 + 1}}{\sqrt{(a_2 - a_1')^2 + 1}}.$$

The trilinear coordinates $\alpha:\beta:\gamma$ of the Nagel point $N_{\mu_{\tilde{A}}}$ for triangle DBE is calculated as follows.

$$\alpha_{\mu_{\tilde{A}}} = \frac{b + e - d}{d} = \frac{(a_3 - a_1) + \sqrt{(a_2 - a_1)^2 + 1} - \sqrt{(a_2 - a_3)^2 + 1}}{\sqrt{(a_2 - a_3)^2 + 1}}.$$

$$\beta_{\mu_{\tilde{A}}} = \frac{e + d - b}{b} = \frac{\sqrt{(a_2 - a_1)^2 + 1} + \sqrt{(a_2 - a_3)^2 + 1} - (a_3 - a_1)}{a_3 - a_1}.$$

$$\gamma_{\mu_{\tilde{A}}} = \frac{d + b - e}{e} = \frac{\sqrt{(a_2 - a_3)^2 + 1} + (a_3 - a_1) - \sqrt{(a_2 - a_1)^2 + 1}}{\sqrt{(a_2 - a_1)^2 + 1}}.$$

Finally, the Cartesian coordinates of the Nagel Point related to the non-membership function $N_{1-\nu_{\tilde{A}}}$ and the membership function $N_{\mu_{\tilde{A}}}$ are produced below.

$$N_{1-\nu_{\tilde{A}}} = \left(\frac{\alpha_{1-\nu_{\tilde{A}}} a a_1' + \beta_{1-\nu_{\tilde{A}}} b a_2 + \gamma_{1-\nu_{\tilde{A}}} c a_3'}{\alpha_{1-\nu_{\tilde{A}}} a + \beta_{1-\nu_{\tilde{A}}} b + \gamma_{1-\nu_{\tilde{A}}} c}, \frac{\beta_{1-\nu_{\tilde{A}}} b}{\alpha_{1-\nu_{\tilde{A}}} a + \beta_{1-\nu_{\tilde{A}}} b + \gamma_{1-\nu_{\tilde{A}}} c}\right),$$

and

$$N_{\mu_{\tilde{A}}} = \left(\frac{\alpha_{\mu_{\tilde{A}}} d a_1 + \beta_{\mu_{\tilde{A}}} b a_2 + \gamma_{\mu_{\tilde{A}}} e a_3}{\alpha_{\mu_{\tilde{A}}} d + \beta_{\mu_{\tilde{A}}} b + \gamma_{\mu_{\tilde{A}}} e}, \frac{\beta_{\mu_{\tilde{A}}} b}{\alpha_{\mu_{\tilde{A}}} d + \beta_{\mu_{\tilde{A}}} b + \gamma_{\mu_{\tilde{A}}} e}\right).$$

The Nagel points $N_{1-\nu_{\tilde{A}}}$ and $N_{\mu_{\tilde{A}}}$ corresponding to the non-membership and membership have to be combined, which enables the proposal of a ranking of TIFNs.

Hence $N_{\tilde{A}} = pN_{\mu_{\tilde{A}}} + qN_{1-\nu_{\tilde{A}}}$, where $p + q = 1$. For simpler ranking, $N_{\tilde{A}} = (x_{\tilde{a}}, y_{\tilde{a}}) = \frac{N_{\mu_{\tilde{A}}} + N_{1-\nu_{\tilde{A}}}}{2}$.

The new ordering of TIFNs using the Nagel point is given by $N(\tilde{A}) = (x_{\tilde{a}}, y_{\tilde{a}}, a_2)$. The proposed ranking procedure is well explained by the forthcoming algorithm.

Algorithm 1. The approach of ranking any two TIFNs is summarized as follows:

Let $\tilde{A} = (a_1, a_2, a_3; a_1', a_2, a_3')$ and $\tilde{B} = (b_1, b_2, b_3; b_1', b_2, b_3')$ be any two TIFNs.

Step 1. Compute $N_{\mu_{\tilde{A}}}, N_{1-\nu_{\tilde{A}}}$

Step 2. Compute $N_{\mu_{\tilde{B}}}, N_{1-\nu_{\tilde{B}}}$

Step 3. Obtain the Nagel points for the two TIFNs $\tilde{A}_{TIFN}$ and $\tilde{B}_{TIFN}$ respectively.

Step 4. Calculate $N(\tilde{A})$ and $N(\tilde{B})$.

Step 5. Then lexicographical ordering is employed to rank the given TIFNs.

I. $\tilde{A} \prec \tilde{B}$ if $N(\tilde{A}) <_L N(\tilde{B})$

II. $\tilde{A} \succ \tilde{B}$ if $N(\tilde{A}) >_L N(\tilde{B})$

III. $\tilde{A} \approx \tilde{B}$ if $N(\tilde{A}) =_L N(\tilde{B})$

In the above ordering, the notation $<_L$ represents the lexicographical ordering defined by $(x_1, x_2, x_3) <_L (y_1, y_2, y_3) \Leftrightarrow (\exists n = 1,2,3)(\forall j < n)(x_j = y_j) \wedge (x_n < y_n)$. To be more specific, $(x_1, x_2, x_3) <_L (y_1, y_2, y_3) \Leftrightarrow (x_1 < y_1)(or)(x_1 = y_1 and x_2 < y_2)(or)(x_1 = y_1, x_2 = y_2 \text{ and } x_3 < y_3)$.

4 | Reasonable Ordering Properties of Proposed Ranking

In [23], Wang and Kerre provide a set of axioms as appropriate characteristics for assessing an order technique's rationality. In this part, it will be demonstrated that the suggested method satisfies Wang and Kerre's reasonable qualities.

Theorem 1. Let $\mathbb{T}$ be the set of all TIFNs, and let E be a subset of $\mathbb{T}$. Then for any $\tilde{A} \in E, \tilde{A} \preccurlyeq \tilde{A}$.

Theorem 2. For any arbitrary finite subset E of $\mathbb{T}$ and $(\tilde{A}, \tilde{B}) \in E \times E$, if $\tilde{A} \preccurlyeq \tilde{B}, \tilde{B} \succcurlyeq \tilde{A}$, we have $\tilde{A} \cong \tilde{B}$.

Theorem 3. For any arbitrary finite subset E of $\mathbb{T}$ and $(\tilde{A}, \tilde{B}) \in E \times E$, if $\tilde{A} \preccurlyeq \tilde{B}, \tilde{B} \preccurlyeq \tilde{C}$, we have $\tilde{A} \preccurlyeq \tilde{C}$.

Theorem 4. Let $\tilde{A}$ and $\tilde{B}$ be any two TIFN's. Then $N_{\tilde{A}+\tilde{B}} = N_{\tilde{A}} + N_{\tilde{B}}$.

5 | Numerical Examples

In this section, an example illustrating the proposed algorithm in the previous section was initiated first. Then a comparative study with existing ordering procedures in the literature was made, and results were tabulated.

Example 1. Consider any two TIFN's $\tilde{A}_{TIFN} = (3,4,5; 2,4,6)$ and $\tilde{B}_{TIFN} = (1,2,3; 0,2,6)$.

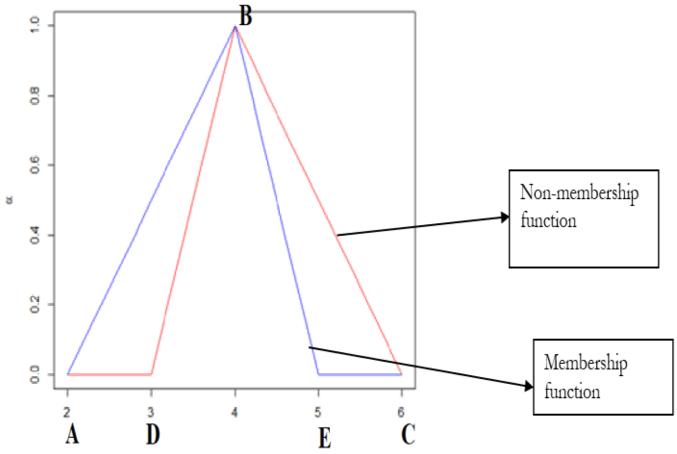

Fig. 1. Representation of $\tilde{A}_{TIFN} = (3, 4, 5; 2, 4, 6)$.

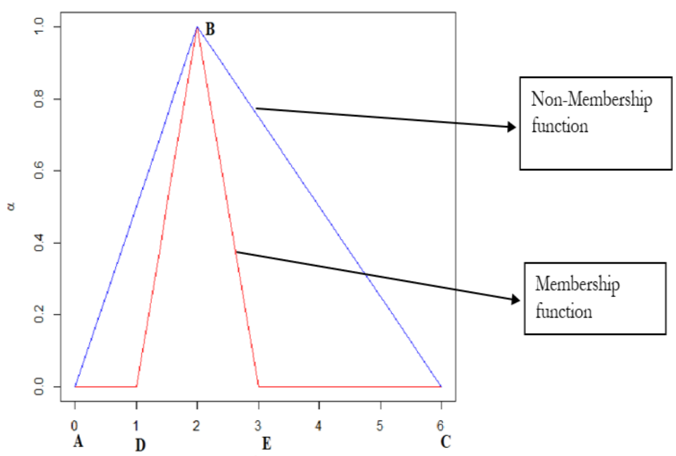

Fig. 2. Representation of $\tilde{B}_{TIFN} = (1, 2, 3; 0, 2, 6)$.

Now

$$\alpha_{1-\nu_{\tilde{A}}} = \frac{6-2+\sqrt{(4-2)^2+1}-\sqrt{(4-6)^2+1}}{\sqrt{(4-6)^2+1}} = 1.7889.$$

$$\beta_{1-\nu_{\tilde{A}}} = \frac{\sqrt{(4-6)^2+1}+\sqrt{(4-2)^2+1}-(6-2)}{4} = 0.1180.$$

$$\gamma_{1-\nu_{\tilde{A}}} = \frac{\sqrt{(4-6)^2+1}+6-2-\sqrt{(4-2)^2+1}}{\sqrt{(4-2)^2+1}} = 1.7889.$$

$$\alpha_{\mu_{\tilde{A}}} = \frac{(5-3)+\sqrt{(4-3)^2+1}-\sqrt{(4-5)^2+1}}{\sqrt{(4-5)^2+1}} = 1.4142.$$

$$\beta_{\mu_{\tilde{A}}} = \frac{\sqrt{(4-3)^2+1}+\sqrt{(4-5)^2+1}-(5-3)}{5-3} = 0.4142.$$

$$\gamma_{\mu_{\tilde{A}}} = \frac{\sqrt{(4-5)^2+1}+(5-3)-\sqrt{(4-3)^2+1}}{\sqrt{(4-3)^2+1}} = 1.4142.$$

Now $N_{1-\nu_{\tilde{A}}} = (4.0012, 0.0557), N_{\mu_{\tilde{A}}} = (4.1248, 0.2553)$.

Then $N_{\tilde{A}} = (x_{\tilde{a}}, y_{\tilde{a}}) = (4.063, 0.156)$. The final ranking is given by $N(\tilde{A}) = (4.063, 0.156, 4)$.

Similarly, calculations show that $N(\tilde{B}) = (2.903, 0.103, 2)$. Since $N(\tilde{A}) >_L N(\tilde{B}) \Rightarrow \tilde{A} > \tilde{B}$.

The comparison between the suggested ordering and the current approaches is displayed in the following table. Our suggested approach is consistent with the current approaches, demonstrating the reliability of the new ranking. Consider the TIFN's $\tilde{A} = (2,3,4; 1,3,5), \tilde{B} = (1,2,3; 0,2,6)$.

Table 1. Comparison of the proposed ranking method with existing approaches for TIFNs.

S.No	Author	Procedure Adopted	Ranking Values	Ordering
1	Bharati [4]	Sign distance of TIFN	$R(\tilde{A}) = 1.5, R(\tilde{B}) = 0.75$	$\tilde{A} > \tilde{B}$
2	Atalik and Senturk [5]	Novel ranking approach based on incircle	$\text{Rank}(A) = (2.778, 0.557, 2)$, $\text{Rank}(B) = (1.803, 0.550, 2)$	$\tilde{A} > \tilde{B}$
3	Arun Prakash et al. [24]	Ranking based on centroid	$R(\tilde{A}) = 2.63, R(\tilde{B}) = 1.9$	$\tilde{A} > \tilde{B}$
4	Mohan et al. [25]	Magnitude ranking	$R(\tilde{A}) = 2.42, R(\tilde{B}) = 1.83$	$\tilde{A} > \tilde{B}$
5	Saini et al. [17]	Distance measure	$R(\tilde{A}) = 2.78, R(\tilde{B}) = 2.2$	$\tilde{A} > \tilde{B}$
6	Nagoorgani and Ponnalagu [23]	Accuracy function	$R(\tilde{A}) = 3, R(\tilde{B}) = 2.25$	$\tilde{A} > \tilde{B}$
7	Proposed method	Ranking based on Nagel Points	$N(\tilde{A}) = (4.063, 0.156, 4)$. $N(\tilde{B}) = (2.903, 0.103, 2)$.	$\tilde{A} > \tilde{B}$

6 | Innovative Ranking Framework for MCDM: A Practical Application

This section presents a multi-criteria decision-making problem utilizing TIFNs as the decision-making quantifiers, based on the suggested ranking approach.

The multi-criteria decision procedure is framed with the criterion $C = (c_1, c_2, \dots, c_n)$ to assess the m alternatives $A = (a_1, a_2, \dots, a_m)$. The best option that maximizes the benefit criterion and minimizes the cost criterion will be chosen by the decision maker.

The weight vector of the criteria is expressed by w_j. The linguistic expression of the expert to represent the criteria C_j in alternative A_i is given by the TIFNs $\tilde{A}_{ij} = (a_{ij}. b_{ij}, c_{ij}; a'_{ij}. b_{ij}, c'_{ij})$. Therefore, the MCDM problem can be constructed in matrix form as

$$D = [\tilde{A}_{ij}]_{m\times n} = \begin{bmatrix} A_{11} & A_{12} & . & . & . & A_{1n} \\ A_{21} & A_{22} & . & . & . & A_{2n} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ A_{n1} & A_{n2} & . & . & . & A_{nn} \end{bmatrix}.$$

Now the detailed step-by-step procedure to solve the constructed MCDM problem is as follows:

Step 1 (compute the normalized TIFNs decision matrix). The primary step to avoid certain mismatches in measures of criteria, such as scaling and comparison, can be neutralized by normalization. The normalized decision matrix has been constructed using the following formula; the normalization of values in the decision matrix can be obtained.

$$v_{ij} = \left\{ \left(\left(\frac{a_{ij}}{a_j^{max} \frac{b_{ij}}{a_j^{max} \frac{c_{ij}}{a_j^{max}}} \left(\frac{a'_{ij}}{a_j^{max} \frac{b_{ij}}{a_j^{max} \frac{c'_{ij}}{a_j^{max}}}} \{\} \right)} \right) \right) \right\}.$$

$$N = [\tilde{v}_{ij}] = \begin{bmatrix} v_{11} & v_{12} & . & . & . & v_{in} \\ v_{21} & v_{22} & . & . & . & v_{2n} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ v_{n1} & v_{n2} & . & . & . & v_{nn} \end{bmatrix}.$$

Step 2 (compute the weighted normalized TIFNs decision matrix). The elements of the weighted normalized TIFNs decision matrix are computed by using the formula

$$V_{ij} = w_j \otimes v_{ij}.$$

The transformed decision matrix is given by

$$W = [V_{ij}] = \begin{bmatrix} V_{11} & V_{12} & . & . & . & V_{1n} \\ V_{21} & V_{22} & . & . & . & V_{2n} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ V_{n1} & V_{n2} & . & . & . & V_{n3} \end{bmatrix}.$$

Step 3. Compute the aggregated values of the attributes

$$R_i = \sum_{j=1}^{n} V_{ij}.$$

Step 4 (now rank the alternatives). Apply the proposed Nagel point ranking for R_i for each alternative and conclude the problem by the ranking procedure.

Step 5. Depending on the alternatives and criteria's nature, conclude the decision by selection of maximum or minimum values.

Case study: This section discusses a real-world example to illustrate the suggested ranking process. Selecting the ideal site for an aerospace research organization center installation is crucial. Five areas were discovered by the space scientists group after a preliminary survey:

A1, A2, A3, A4, and A5. Five criteria were used by the space scientists group to evaluate the locations: Geographic position (C1), climatic condition (C2), safety factor (C3), functional area (C4), and pollution factor (C5).

Let w = $(0.2, 0.15, 0.25, 0.1, 0.3)^T$ represent the criterion weight factor. The following table shows how the alternatives were rated using the specified criteria.

Table 1. TIFN's decision matrix for five alternatives.

	Criteria				
Alternatives	**C1**	**C2**	**C3**	**C4**	**C5**
A1	(1,2,3 ; 0,2,4)	(3,4,5;1,4,5)	(0,2,5;1,2,4)	(4,5,7; 3,5,8)	(4,5,6;3,5,7)
A2	(2,3,4;1,3,5)	(6,7,8;5,7,9)	(4,5,6; 2,5,7)	(3,4,5;1,4,6)	(6,7,8;4,7,9)
A3	(1,2,3;0,2,5)	(4,6,7;3,6,8)	(3,4,5;0,4,6)	(4,5,6;1,5,7)	(5,6,7 ;3,6,8)
A4	(2,3,4;1,3,6)	(5,6,7 ;2,6,8)	(2,3,5;1,3,6)	(3,4,5;2,4,7)	(4,6,7;2,6,8)
A5	(2,3,4;0,3,5)	(4,5,6;1,5,7)	(2,4,5;1,4,6)	(3,5,7;2,5,8)	(3,5,6 ; 2,5,7)

Table 2. Normalized TIFNs decision matrix for five alternatives.

	Criteria				
Alternatives	**C1**	**C2**	**C3**	**C4**	**C5**
A1	(0.2,0.3,0.5;0,0.3,0.7)	(0.3,0.4,0.6;0.1,0.4,0.6)	(0,0.3,0.7;0.1,0.3,0.6)	(0.5,0.6,0.9;0.4,0.6,1.0)	(0.4,0.6,0.7;0.3,0.6,0.8)
A2	(0.3,0.5,0.7;0.2,0.5,0.8;)	(0.7,0.8,0.9;0.6,0.8,1.0)	(0.6,0.7,0.9;0.3,0.7,1.0)	(0.4,0.5,0.6;0.1,0.5,0.8)	(0.7,0.8,0.9;0.4,0.8,1.0)
A3	(0.2,0.3,0.5;0,0.3,0.8)	(0.4,0.7,0.8;0.3,0.7,0.9)	(0.4,0.6,0.7;0,0.6,0.9)	(0.5,0.6,0.8;0.1,0.6,0.9)	(0.6,0.7,0.8;0.3,0.7,0.9)
A4	(0.3,0.5,0.7;0.2,0.5,1.0)	(0.6,0.7,0.8;0.2,0.7,0.9)	(0.3,0.4,0.7;0.1,0.4,0.9)	(0.4,0.5,0.6;0.3,0.5,0.9)	(0.4,0.7,0.8;0.2,0.7,0.9)
A5	(0.3,0.5,0.7;0,0.5,0.8)	(0.4,0.6,0.7;0.1,0.6,0.8)	(0.3,0.6,0.7;0.1,0.6,0.9)	(0.4,0.6,0.9;0.3,0.6,1.0)	(0.3,0.6,0.7;0.2,0.6,0.8)

Table 3. Weighted Normalized TIFNs decision matrix for five alternatives.

	Criteria				
Alternatives	**C1**	**C2**	**C3**	**C4**	**C5**
A1	(0.03,0.07,0.10;0.00,0.07,0.13)	(0.05,0.07,0.08;0.02,0.07,0.08)	(0,0.07,0.18;0.04,0.07,0.14)	(0.05,0.06,0.09;0.04,0.06,0.10)	(0.13,0.17,0.20;0.10,0.17,0.23)
A2	(0.07,0.10,0.13;0.03,0.10,0.17)	(0.10,0.12,0.13;0.080.12,0.15)	(0.14,0.18,0.21;0.07,0.18,0.25)	(0.04,0.05,0.06;0.01,0.05,0.08)	(0.200.23,0.27;0.13,0.23,0.30)
A3	(0.03,0.07,0.10;0.00,0.07,0.17)	(0.07,0.10,0.12;0.05,0.10,0.13)	(0.11,0.14,0.18;0.0,0.14,0.21)	(0.05,0.06,0.08;0.01,0.06,0.09)	(0.17,0.20,0.23;0.10,0.20,0.27)
A4	(0.07,0.10,0.13;0.03,0.10,0.20)	(0.08,0.10,0.12;0.03,0.10,0.13)	(0.07,0.11,0.18;0.04,0.11,0.21)	(0.04,0.05,0.06;0.03,0.05,0.09)	(0.13,0.20,0.23;0.07,0.20,0.27)
A5	(0.07,0.10,0.13;0.0,0.10,0.17)	(0.07,0.08,0.10;0.02,0.08,0.12)	(0.07,0.14,0.18;0.04,0.14,0.21)	(0.04,0.06,0.09;0.03,0.06,0.10)	(0.10,0.17,0.20;0.07,0.17,0.23)

Table 4. Final decision results.

Alternatives	Aggregated Values
A1	(0.27,0.43,0.65;0.19,0.43,0.69)
A2	(0.55,0.68,0.81;0.33,0.68,0.94)
A3	(0.42,0.57,0.70;0.16,0.57,0.87)
A4	(0.39,0.56,0.72;0.19,0.56,0.90)
A5	(0.34,0.56,0.70;0.14,0.56,0.83)

Applying the Nagel point concept, the final decision can be obtained as, $A_2 > A_3 > A_4 > A_5 > A_1$Comparative ranking with existing methods

Table 5. Ranking using different methods.

Methods	Results
Li	$A_2 > A_3 > A_4 > A_5 > A_1$
Dubey and Mehra	$A_2 > A_3 > A_4 > A_5 > A_1$
Wei	$A_2 > A_3 > A_4 > A_5 > A_1$
Rezvani	$A_2 > A_4 > A_3 > A_5 > A_1$
Wang and Zhong	$A_3 > A_2 > A_4 > A_5 > A_1$
Das and Guha	$A_2 > A_3 > A_4 > A_5 > A_1$
Proposed method	$A_2 > A_3 > A_4 > A_5 > A_1$

The above table shows that the proposed ranking method aligns with the existing ranking methods.

7 | Conclusion

This article suggests a new ordering of TIFNs using Nagel points as a basis. Some numerical examples to illustrate the applicability of the strategy are provided. Moreover, a comparison analysis is carried out to verify the proposed methodology. A comparison with a few other methods in use is used to demonstrate the ranking method's superiority. This research has some theoretical and managerial ramifications.

In conclusion, the process of ranking TIFNs is a complex and crucial task, given the inherent uncertainty and imprecision associated with these numbers. The study aimed to develop a systematic approach for ranking TIFNs, taking into account both the membership and non-membership degrees, to enhance decision-making processes in various real-world applications.

Through an extensive review of existing methods and a critical evaluation of their limitations, this research proposed a novel ranking method based on a comprehensive consideration of the centroid deviation and area deviation between TIFNs. The proposed method successfully addresses the challenges posed by existing approaches, offering a more robust and accurate means of ranking TIFNs.

The significance of this research extends to diverse fields where decision-making involves uncertainty and imprecision. Whether in finance, engineering, or other domains, the ability to rank TIFNs accurately contributes to more informed and confident decision-making processes. The proposed method not only fills a gap in existing methodologies but also opens avenues for further research and refinement in the realm of ranking under uncertainty.

In conclusion, the developed ranking method for TIFNs offers a valuable contribution to the field, enhancing our ability to navigate decision-making in situations characterized by vagueness and ambiguity. As decision-makers continue to grapple with complex, uncertain information, the proposed method provides a reliable tool for ranking TIFNs, promoting more effective and robust decision outcomes.

Declaration of Competing Interest

The authors declare that they have no conflicts of interest in this work. We declare that we do not have any commercial or associative interest that represents a conflict of interest in connection with the work submitted. Compliance with ethical standards. Also, we assure that we have not received any funding support.

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