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Economies of Scope in Network-Structured Production Systems

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Abstract

Methods for determining economies of scope in Data Envelopment Analysis (DEA) have been widely developed to investigate and examine the effects of product or service diversification on production costs. However, these techniques are not sufficiently efficient for analyzing economies of scope in systems with network structures, including network-structured production systems. The major reason for this inefficiency is the neglect of the internal structure of production units and the disregard of their intermediate products. Therefore, in this paper, while introducing the concept of economies of scope and reviewing the conventional DEA-based models for its calculation, a new method is proposed which, by taking into account the internal structure of units, is capable of evaluating the performance of virtual diversified firms formed through the integration of specialized firms with network structures. One of the important features of the proposed method is that it creates an opportunity for cooperation between the analyst and the decision-maker (manager). This cooperation facilitates the adoption of appropriate decisions aimed at reducing production costs, such as the merger of specialized firms for the joint production of products and services, or the elimination of a product from the production cycle in order to specialize the production process. By considering intermediate products, the proposed approach performs more accurately in comparison with conventional DEA methods, and planning based on it is consequently more reliable. The inefficiency of traditional DEA approaches regarding this concept is demonstrated through a numerical example in comparison with the proposed method.

Keywords: Data envelopment analysis, Network-structured production systems, Intermediate products, Economies of scope.

1 | Introduction

Economies of scope constitute one of the most important factors influencing the performance of production units and reducing production costs, thereby enhancing the competitive power of firms. Economies of scope arise when cost savings can be achieved as a result of the joint production of products and services. In fact,

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this concept reflects the effect of product diversification on production costs and, due to its undeniable impact on the performance of Decision-Making Units (DMUs), enables firms to gain market dominance. In a competitive market, if a production unit is able to manufacture its products at lower costs, it can offer them to the market at lower prices and consequently capture a larger market share [1]. Panzar and Willig [2], as well as Baunol et al. [3], introduced the general criterion for economies of scope. This criterion represents the cost savings resulting from the simultaneous production of two products compared with producing them separately.

Färe [4] extended the work of Baunol et al. [3] by employing Data Envelopment Analysis (DEA). DEA, first introduced by Charnes et al. [5], is a comprehensive approach widely accepted for evaluating the performance of homogeneous DMUs, and it can also be applied to analyze the concept of economies of scope. Most studies in this field are based on the idea proposed by Färe [4]. All models developed by researchers for this purpose have considered production firms as black boxes, whereas in reality most production systems possess network structures.

Therefore, under such conditions, there is a need for models capable of evaluating economies of scope while taking into account intermediate products and the internal structure of these units [6]. The objective of this paper is to fill the existing gap in studies concerning the evaluation and analysis of economies of scope and to extend this concept to network-structured (two-stage) production systems by demonstrating the effect of intermediate products through the application of Network Data Envelopment Analysis (NDEA) techniques.

2 | Economies of Scope Based on Data Envelopment Analysis

Economies of scope are generally defined as factors that make the joint production of a set of products less costly than producing them separately. This concept arises from centralized and shared business capabilities such as finance and marketing. These factors can also be identified within business processes and operations, for example through the implicit cross-selling of one product alongside another[1].

Therefore, in order to study and examine the effect of product diversification on production costs, the concept of economies of scope is employed. In other words, this concept refers to the cost savings resulting from producing a wide range of products or providing diverse services.

Panzar and Willig [2] introduced the term "economies of scope" to describe a fundamental and appealing characteristic of production theory, considering it an indicator of the cost advantages associated with the joint production of different products and services. Since its introduction, this concept has been widely used as an important economic indicator in evaluating the performance of diversified firms. Several methods have been proposed for measuring this economic indicator.

In the present study, the nonparametric DEA approach is employed to directly compare the costs of joint production and specialized production of products in network-structured production systems [6]. This important economic indicator, through an accurate evaluation of the performance of diversified firms in comparison with specialized firms, both operating within network structures, can provide managers with valuable support for future decisions regarding the retention or elimination of product lines.

Baumol et al. [3], while defining diversified and specialized firms, stated that economies of scope between two products exist when the cost of their simultaneous production within a single firm is lower than the cost of producing the same products separately in specialized firms[2].

Mathematically, they presented the following definition:

[1] McDonald's applies this principle in the production of products such as burgers, French fries, sandwiches, and salads.

[2] Production firms that simultaneously produce multiple products are referred to as diversified firms, whereas firms that produce only one of those products are called specialized firms.

2.1 | Economies of Scope

There will be economies of scope between the two outputs y_1 and y_2 if:

$$c(y_1, y_2) < c(y_1, 0) + c(0, y_2), \tag{1}$$

where $c(y_1, y_2)$ denotes the joint production cost of two products in a multiproduct firm, while $c(y_1, 0)$ and $c(0, y_2)$ respectively represent the costs of producing y_1 and y_2 separately in specialized firms [3]. If the reverse of *Eq. (1)* holds, meaning that the separate production cost of the products in different firms is lower than the production cost within a multiproduct firm, diseconomies of scope occur.

It should be noted that *Eq. (1)* is a necessary but not sufficient condition for the existence of economies of scope. A sufficient condition is the presence of a complementary cost relationship between the two outputs; that is, adding the second product to the production cycle reduces the marginal cost of producing the first product. In fact, the existence of a complementary relationship between the costs of the two outputs indicates the advantages of joint production.

Another indicator introduced by Baunol et al. [3] for measuring economies of scope is the Degree of Economies of Scope (DES), which is scale-independent and reflects the percentage of cost savings achieved through the simultaneous production of products within a multiproduct firm.

2.2 | Degree of Economies of Scope

The DES for a firm is defined as follows:

$$DES_j = \frac{C(y_1, 0) + C(0, y_2) - C(y_1, y_2)}{C(y_1, y_2)}.$$

According to *Eq. (2)*, if $DES > 0$, the firm's multiple-product production is considered to exhibit economies of scope. A positive value of this index indicates that joint production is cost-efficient and reduces production costs. When $DES < 0$, diseconomies of scope exist, indicating that specialized production of the products in separate firms is more cost-efficient. If $DES = 0$, the cost of joint production equals the cost of separate production in specialized firms, implying that no cost advantage exists for either type of firm.

To investigate economies of scope within the DEA framework, suppose that the set D consists of n diversified firms, each using one input (cost) to produce two outputs. Let the vectors c_j and $y_j = (y_{1j}, y_{2j})$ denote the cost vector and the output vector, respectively, for diversified firm j. The production possibility set (technology set) of this group includes all possible cost combinations capable of producing the outputs of the diversified firms under consideration. Therefore, under the assumption of Constant Returns to Scale (CRS), this set is defined as follows [6]:

$$P_{1,2} = \left\{(c, y) \mid c = \sum\nolimits_{j=1}^{n} c_j\, \mu_j,\ y \le \sum\nolimits_{j=1}^{n} y_j\, \mu_j,\ \mu \ge 0\right\}.$$

Now suppose that S_1 and S_2 respectively consist of p and q specialized firms that use the input (cost) to produce only one of the two products. The specialized firms in group S_1 produce only output y_{1j} using cost v_{1j}, for $j = 1, \dots, p$, while the specialized firms in group S_2 produce only output y_{2j} using input (cost) v_{2j}, for $j = 1, \dots, q$. The production possibility set for these two groups of specialized firms can also be expressed as follows:

$$P_1 = \left\{(v, y_1) \mid v = \sum\nolimits_{j=1}^{p} v_{1j}\, \mu_j,\ y_1 \le \sum\nolimits_{j=1}^{p} \hat{y}_{1j}\, \mu_j,\ \forall j{:}\ \mu_j \ge 0\right\}, \tag{3}$$

$$P_2 = \left\{(v, y_2) \mid v = \sum_{j=1}^{q} v_{2j}\, \mu_j,\ y_2 \le \sum_{j=1}^{q} \hat{y}_{2j}\, \mu_j,\ \forall j{:}\ \mu_j \ge 0\right\}.$$

Now let M denote the set of all virtual diversified firms formed by combining p specialized firms from group S_1 and q specialized firms from group S_2. To calculate the efficient frontier of diversified firms, the CCR model (*Model (3)*) is first applied to the specialized firms in groups S_1 and S_2 [7]:

$$\begin{aligned} &\min\ \theta_o \\ &\text{s.t}\ \sum_{j=1}^{n} \mu_j x_{ij} \le \theta_o x_{ik}, && i = 1, \dots, m, \\ &\sum_{j=1}^{n} \mu_j y_{rj} \ge y_{rk}, && r = 1, \dots, s, \\ &\mu_j \ge 0, && j = 1, \dots, n. \end{aligned} \tag{3}$$

Thus, the minimum production cost for each specialized firm in these two groups can be obtained as follows:

$$v_{1k}^* = \theta_k^* v_{1k}, \mathrm{DMU}_k \in S_1, k = 1, \dots, p$$
$$v_{2h}^* = \theta_h^* v_{1h}, \mathrm{DMU}_h \in S_2, h = 1, \dots, q$$

It should be noted that θ_k^* and θ_h^* are the optimal values of the CCR model for evaluating unit k from the specialized group S_1 and unit h from group S_2, respectively. Furthermore, the input and output vectors of each virtual diversified unit in set M are respectively defined as $v_k = v_{1k}^* + v_{2h}^*$ and $\begin{pmatrix}\hat{y}_{1k}\\ \hat{y}_{2h}\end{pmatrix}$. Therefore, the set of virtual diversified firms M and the corresponding production possibility set are expressed as follows:

$$M = \{(v_j, \hat{y}_{1j}, \hat{y}_{2j}) \mid j = 1, \dots, pq\},$$

$$P_M = \left\{(v, y_1', y_2') \mid v \ge \sum_{j=1}^{n} c_j\, \mu_j,\ y' \le \sum_{j=1}^{n} y_j\, \mu_j,\ \forall j{:}\ \mu_j \ge 0\right\}.$$

To examine the existence of economies of scope for each firm in the diversified production possibility set $(P_{1,2})$, Kuosmanen et al. [8] proposed the following model:

$$\begin{aligned} &\min\ \theta \\ &\text{s.t.}\ v_k\theta \ge \sum_{j=1}^{n} \mu_j\, c_j, \\ &y'_{rk} \le \sum_{j=1}^{n} \mu_j\, y_{rj}, && r = 1, 2, \\ &\mu_j \ge 0, && j = 1, \dots, n. \end{aligned} \tag{4}$$

According to *Model (4)*, the minimum joint production cost of products in virtual diversified firm j is given by:

$$c(y_1, y_2) = \theta^* v_j, j = 1, \dots, pq,$$

where θ^* is the optimal value of *Model (4)* for evaluating unit $j$among the firms in set M. It is evident that a firm exhibits economies of scope if:

$$\theta^* < 1.$$

3 | Analysis of Economies of Scope in Network-Structured Production Systems

In real-world applications, many systems under evaluation possess a network structure. As previously mentioned, traditional DEA methods treat such systems as black boxes, considering only the initial inputs and final outputs while ignoring the internal structure of the units in their calculations [9]. Consequently, the results obtained from these models lack sufficient accuracy, making planning and decision-making based on them unreliable.

Therefore, NDEA approaches have been employed for a more accurate evaluation of such systems, since these models provide greater precision and clearly capture the effects of intermediate products on the performance of the units [10]. Although numerous studies in recent years have focused on evaluating the performance of network-structured production systems, a major gap in these studies is the lack of consideration of the economies of scope index, which is widely used in assessing the performance of diversified production units.

In this section, by extending the models presented in Section 2, models are proposed to investigate the impact of product diversification on production costs in network-structured production systems. These models not only determine the effects of intermediate products on the joint production cost of goods and on the separate costs of specialized production, but also identify the more appropriate production strategy, joint production or specialized production, for manufacturers. To estimate the DES in serial network structures, two-stage serial production systems are considered as the base framework.

Two-stage serial production systems are a special type of serial network systems in which two processes are connected sequentially through intermediate products. In such systems, intermediate products play a dual role, acting as the outputs of the first stage and simultaneously as the inputs of the second stage [10].

Most studies conducted in the field of network structures have focused on serial network systems with a two-stage structure. Based on the above explanations, *Fig. 1* illustrates a two-stage serial production system in which z_{dk}, $d = 1, \dots, D$, denotes the d-th intermediate product of unit k.

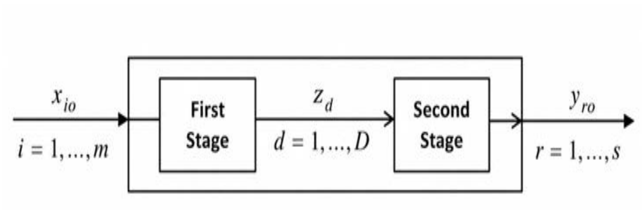

Fig. 1. Two-stage serial production system with intermediate products.

Consider three groups of production firms with a two-stage structure, as illustrated in *Fig. 2*. One group of these firms simultaneously produces both outputs y_1 and y_2 (diversified firms), while the other group is specialized production firms, such that p firms produce only the final product y_1 (group S_1) and q firms produce only the final product y_2 (group S_2).

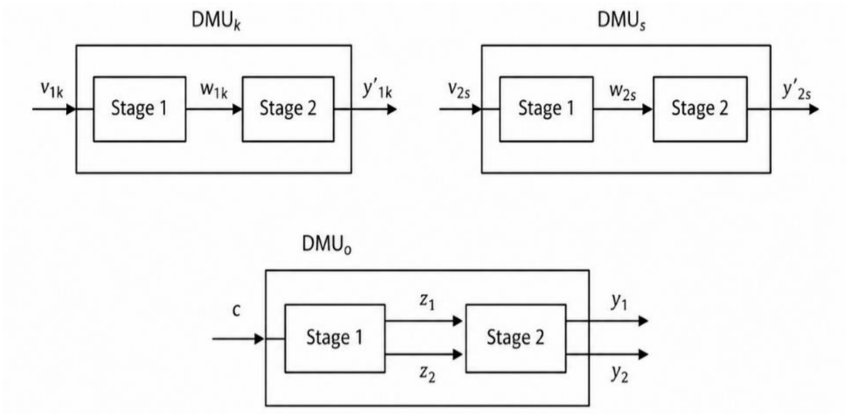

Fig. 2. Three groups of two-stage production firms with diversified and specialized outputs.

It should be noted that each diversified firm includes one input (cost) and two intermediate products, z_1 and z_2, while the intermediate products of the specialized firms in groups S_1 and S_2 are only w_1 and w_2, respectively. The input of all firms is homogeneous and of the cost type, whereas the types of intermediate and final outputs differ among the specialized firms. Furthermore, the intermediate and final outputs of diversified and specialized firms are assumed to be homogeneous.

To estimate and analyze economies of scope in two-stage production systems, it is sufficient to construct virtual diversified firms, similar to Section 2.2, by combining $p$specialized firms from group S_1 with q specialized firms from group S_2, and then evaluate them within the production possibility set generated by the diversified firms.

For this purpose, it is necessary to calculate the production cost of each specialized firm in groups S_1 and S_2. Since the specialized firms themselves possess a two-stage structure, NDEA models for two-stage systems are employed for a more accurate evaluation.

One of the most widely used models for evaluating two-stage production systems is the model proposed by Kao and Hwang [9], which is formulated as follows:

$$\begin{aligned}
&\min \quad \theta \\
&\text{s.t.} \ \sum_{j=1}^{n} \mu_j x_{ij} \ \le \ \theta\, x_{ik}, \quad i = 1, \dots, m. \\
&\sum_{j=1}^{n} \gamma_j y_{rj} \ge \ y_{rk}, \quad r = 1, \dots, s, \\
&\sum_{j=1}^{n} (\mu_j - \gamma_j) z_{dj} \ge 0, \ \ d = 1, \dots, D, \\
&\mu_j, \gamma_j \ge 0, \quad j = 1, \dots, n.
\end{aligned} \tag{5}$$

By evaluating the specialized firms using *Model (5)*, and by defining $v_j = v_{1k}^* + v_{2s}^*$ for $j = 1, \dots, pq$, the input cost of each virtual diversified firm can be obtained. It should be noted that v_{1k}^* and v_{2s}^* respectively denote the costs of $DMU_k \in S_1$ and $DMU_s \in S_2$. Since the types of intermediate and final outputs differ across the specialized firms in the two groups, each virtual diversified firm constructed from them contains one input v_{ks}, two intermediate products $w_{ks} = \binom{w_k}{w_s}$ and two final outputs $\binom{\acute{y}_1 = y_{1k}}{\acute{y}_2 = y_{2s}}$. *Fig. 3* illustrates a virtual diversified firm.

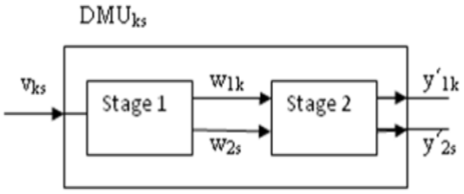

Fig. 3. Virtual diversified firm.

Now, by considering *Model (6)*, each of the virtual units constructed from the specialized firms can be evaluated within the production possibility set of the diversified firms, and the economies of scope for each of them can be calculated.

$$
\begin{aligned}
&\min \quad \theta_o,\\
&\text{s.t.} \quad v_o\theta_o \geq \sum_{j=1}^{n} \mu_j\, c_j,\\
&y'_{ro} \leq \sum_{j=1}^{n} \gamma_j\, y_{rj}, \quad r = 1, 2,\\
&0 \leq \sum_{j=1}^{n} (\mu_j - \gamma_j) z_{dj}, \qquad d = 1, 2,\\
&\mu_j, \gamma_j \geq 0, \qquad j = 1, \dots, n.
\end{aligned} \tag{6}
$$

In *Model (6)*, v_o and $\acute{y}_o$ respectively represent the input and output of the virtual unit under evaluation. The right-hand side of each constraint in *Model (6)* represents the inputs, outputs, and intermediate products of the diversified firms. It should be noted that although the virtual firms include intermediate products, these products have been eliminated from *Model (6)* due to the existence of the following relationships.

$$
w_{do} \leq \sum\nolimits_{j=1}^{n} \mu_j z_{dj}, \tag{1}
$$

where w_{do} denotes the intermediate product of the virtual two-stage unit. Using the following proposition, the desirability of merging production units can be determined.

Proposition 1. Suppose that θ_o^* is the optimal value of *Model (6)* in evaluating the virtual unit $DMU_o \in M$. Then:

I. If $\theta_o^* > 1$, production in specialized firms is more economical.

II. If $\theta_o^* < 1$, the joint production of products in a diversified firm is more economical.

III. If $\theta_o^* = 1$, the cost of producing the products separately in specialized firms is equal to the cost of their joint production in the diversified firm.

Proof: Assume that θ_o^* is the optimal value of *Model (6)* in evaluating the virtual unit $DMU_o \in M$. In this case, the minimum cost of jointly producing the two products in this virtual unit is obtained as follows:

$$
c(y_1, y_2) = \theta_o^*(v_{1k}^* + v_{2s}^*), o \in \{1, \dots, pq\},
$$

and since $v_{1k}^* = c(y_1, 0)$and $v_{2s}^* = c(0, y_2)$ are respectively the minimum (cost-efficient) costs of producing only output $y_1$by DMU_kin $S_1$and the minimum cost of producing only output $y_2$by DMU_kinS_2, we have:

$$
c(y_1, y_2) = \theta_o^*\big(c(y_1, 0) + c(0, y_2)\big).
$$

By considering the last relation, the desirability of merging production units can be examined under the following conditions:

I. If $\theta_o^* > 1$, then merging the production units for joint product manufacturing is not cost-efficient.

II. If $\theta_o^* = 1$, there is no cost saving in joint production compared with separate production of the products.

III. And if $\theta_o^* < 1$, this implies that joint production is cost-efficient.

By considering *Proposition 1* and the definition of DES, the DES can also be calculated using the following relation:

$$\mathrm{DES} = \frac{1}{\theta_o^*} - 1.$$

It should be noted that by extending the above relation, economies of scope in serial network systems can be determined. For this purpose, the virtual diversified firms constructed from combinations of specialized firms with a serial structure obtained from the production possibility set of diversified firms with a network structure is evaluated [10].

In the following, a numerical example is presented to demonstrate the effectiveness of the proposed models in two-stage production systems, which may help provide a better understanding of the subject. The example presented consists of three groups of firms. Two groups of firms produce a single product; in other words, their production is specialized. Unlike these two groups, the third group of firms jointly produces both products. It is also assumed in the presented example that all firms possess a two-stage structure.

4 | Numerical Example

To explain the operation of the algorithm, a simple example involving eleven production firms is used. Among these firms, the two specialized groups each consist of three firms, while the diversified group consists of five firms. Each firm in the specialized groups includes one input, one intermediate product, and one output. The input type is identical in both groups and is cost, whereas the intermediate products and outputs of the two specialized groups are completely different from each other.

In addition, the firms in the diversified group include one input (cost), two intermediate products, and two final outputs. *Table 1* presents the input data, intermediate products, and corresponding outputs of these firms.

Table 1. Input data, intermediate products, and outputs of the hypothetical units.

	DMUs	Input (Cost)	Intermediate Product (z_1)	Intermediate Product (z_2)	Output (y_1)	Output (y_2)
First group of specialized firms	1	4	5	-	4.5	-
	2	6	5	-	7	-
	3	1.5	2	-	3	-
Second group of specialized firms	1	2	-	1.5	-	1.5
	2	5	-	4	-	5.5
	3	3	-	2	-	3
Diversified firms group	1	7	4	4	5	6
	2	6	4	2	4	3
	3	13	5	5.5	7	6.5
	4	16	9	7.5	9.5	10.5
	5	9	1	5.5	1.5	7.5

Now, using the proposed method and the algorithm for calculating the optimal cost of each of the specialized firms in groups S_1 and S_2, nine virtual units are constructed by applying *Model (5)*. *Table 2* presents these costs.

Table 2. Optimal costs of specialized firms.

DMUs	v^*_{1k}	v^*_{2h}
1	0.5625(4)=2.25	0.625(2)=1.25
2	0.5833333(6)=3.4999998	0.9166667(5)=4.5833335
3	1(1.5)=1.5	0.8333333(3)=2.4999999

In the next step of the algorithm, by considering $v_j = v^*_{1k} + v^*_{2s}$ for $j = 1, \dots, 9$, the input costs of the virtual firms are obtained. By applying *Models (6)* and *(7)*, these virtual units constructed from specialized firms in groups S_1 and S_2 are evaluated within the production possibility set of the diversified firms, under both the black-box and two-stage approaches. The results of these evaluations are presented in *Table 3*.

Table 3. Comparison of economies of scope estimates under the black-box and two-stage approaches.

Virtual Firms	θ^* Black Box	θ^* Two Stage	DES Black Box	DES Two Stage	ES Black Box	ES Two Stage
$S_{1,1}$-$S_{2,1}$	0.9712879	0.7108844	0.02956	0.40670	Yes	Yes
$S_{1,1}$-$S_{2,2}$	0.8850575	0.9076284	0.12987	0.10177	Yes	Yes
$S_{1,1}$-$S_{2,3}$	0.9207862	0.7719299	0.08603	0.29545	Yes	Yes
$S_{1,2}$-$S_{2,1}$	1	0.7368421	0	0.35714	Indifferent	Yes
$S_{1,2}$-$S_{2,2}$	0.9088584	0.8006873	0.10028	0.24893	Yes	Yes
$S_{1,2}$-$S_{2,3}$	0.9533482	0.6944445	0.04893	0.44000	Yes	Yes
$S_{1,3}$-$S_{2,1}$	0.9420290	0.7212121	0.06154	0.38655	Yes	Yes
$S_{1,3}$-$S_{2,2}$	0.9871795	1.019528	0.01299	-0.01915	Yes	No
$S_{1,3}$-$S_{2,3}$	0.8920056	0.8513594	0.12107	0.17459	Yes	Yes

In *Table 3*, the decision-making unit $S_{1,k} - S_{2,h}$ represents a virtual firm formed by combining the k-th specialized firm from group S_1 with the h-th specialized firm from group S_2. The results in *Table 3* are examined from two perspectives: the black-box approach and the two-stage approach. A comparison of these two perspectives reveals interesting findings regarding ES and DES, as well as the existence or absence of economies of scope for a particular firm.

The results obtained for the virtual firms $S_{1,2} - S_{2,1}$ and $S_{1,3} - S_{2,2}$ indicate that the two approaches do not necessarily yield identical conclusions. The evaluation of firm $S_{1,3} - S_{2,2}$ under the black-box approach shows that this firm exhibits economies of scope, whereas its evaluation under the two-stage system indicates diseconomies of scope. Likewise, the analysis of the virtual firm $S_{1,2} - S_{2,1}$ under the black-box approach shows that the cost of joint production of the outputs is equal to the cost of producing them separately, implying that merging the specialized firms does not affect reducing or increasing production costs. However, when the same unit is analyzed under the two-stage approach, the cost of joint production is lower than the cost of separate production. Therefore, under this approach, the firm exhibits economies of scope. The remaining virtual units constructed show identical economies of scope results under both approaches. Furthermore, the presence or absence of intermediate products also affects the calculation of DES for the virtual units, as clearly demonstrated by the results presented in the table.

It should be noted that the effect of ES on managerial and producer decisions is based on the preference for joint production of products and services over their specialized production. This issue can also be analyzed from another perspective based on the presence or absence of intermediate products in the evaluation of economies of scope. According to *Table 3*, if the intermediate products in unit $S_{1,3} - S_{2,2}$ are ignored, then $\mathrm{DES}^{\mathrm{Blackbox}}_{S_{1,3}-S_{2,2}} > 0$, indicating that the cost of joint production of products in a diversified firm is lower. In this case, managers and producers will choose a diversified firm. However, if the same firm is analyzed under the two-stage approach, economies of scope in separate production become preferable because:

$$\mathrm{DES}^{\mathrm{Two\ stage}}_{S_{1,3}-S_{2,2}} < 0.$$

Thus, specialized firms are selected for product manufacturing, contrary to the decision derived from the black-box approach. Based on the values of θ^* and DES obtained for firm $S_{1,3} - S_{2,2}$ under the two-stage approach, it is clear that the cost of jointly producing the outputs in this virtual diversified firm is $\frac{1}{9}$% higher than the cost of producing them separately in the specialized firms (three firms from group $S_1$and two firms from group S_2). Therefore, selecting specialized firms would be more beneficial for managers and producers. In this case, a diversified firm would be selected as the production company only if the production costs in the specialized firms increased by more than 6/20212884. In other words, if the costs of specialized production of the outputs increased by 1/9%, then choosing the diversified firm would become economically efficient.

5 | Conclusion

In this paper, the concept of economies of scope was introduced as an influential source affecting the performance of production units through the reduction of production costs. This important economic indicator evaluates the performance of diversified firms by studying and examining the impact of product diversification on production costs. For this purpose, virtual diversified firms constructed from the combination of two different specialized firms were evaluated within the production possibility set of diversified firms. By extending this concept to network-structured production systems, particularly two-stage structures, a model was proposed that, by considering the internal structure of the units, is capable of evaluating the performance of diversified firms with a two-stage structure.

An important feature of the proposed method is that it creates an opportunity for cooperation between the analyst and the decision-maker in order to make appropriate decisions aimed at reducing production costs. The analyses clearly demonstrated that intermediate products play a significant role in the existence or absence of economies of scope within the units. Furthermore, the obtained results play a fundamental role in guiding the future strategic decisions of managers and producers. The numerical examples presented indicate the superiority of the proposed method compared with conventional DEA approaches.

Conflict of Interest

The authors certify that they have no affiliations with or involvement in any organization or entity with any financial or non-financial interest in the subject matter discussed in this manuscript.

Data Availability

All data relevant to this study are contained within the manuscript. Additional information is available from the corresponding author upon reasonable request.

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